

Wideband "Brewster" Angle

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Abstract—This paper investigates the theoretical design and physical behavior of optical thin films under uniform transverse electromagnetic fields. By defining the complex scalar potentials and solving Maxwell's equations across three distinct boundaries, exact analytical expressions for the reflection and transmission coefficients are derived for both TE and TM polarizations. A key focus is determining the optimal conditions to minimize total reflected power in the visible spectrum. The analytical model is supported by computational MATLAB simulations, providing contour plots of reflectivity and line plots of electric field magnitude distributions. These visualizations confirm the mathematical findings and illustrate distinct physical phenomena, including the Meissner-like field exclusion in metals and Fabry-Pérot-like interference in dielectrics.

Index Terms—Maxwell's Equations, Thin-Film Optics, TE Polarization, TM Polarization, Reflection Coefficient, Electric Field Distribution, Interference, Visible Spectrum.

I. REFLECTION COEFFICIENT FOR THE TWO LINEAR POLARIZATIONS

To find the reflection coefficient for the two linear polarizations (TM and TE), we must solve Maxwell's equations in the three regions (before the layer, inside the layer, and after the layer) and apply the boundary conditions at the interfaces $z = 0$ and $z = h$. Let medium 1 be air (or vacuum) with permittivity $\epsilon_1 = \epsilon_0$ and magnetic permeability $\mu_1 = \mu_0$. The layer has thickness h and complex dielectric constant $\epsilon_2(\omega) = \epsilon_0 n^2(\omega)$, where $n(\omega) = n_r(\omega) + ik(\omega)$ is the complex refractive index of the material (it depends on the frequency ω and thus on the wavelength λ). The magnetic permeability of the layer is $\mu_2 = \mu_0$. Medium 3 is also air (or vacuum) with $\epsilon_3 = \epsilon_0$ and $\mu_3 = \mu_0$. The incident beam has an angle of incidence $\theta_i = \theta$ with respect to the z -axis. Due to the homogeneity in the x and y directions, the wave vector component parallel to the surface is conserved in all regions (Snell's law for complex refractive indices).

For TE polarization (the electric field is parallel to the x -axis, $E = (E_x, 0, 0)$), the reflection (r_{TE}) and transmission (t_{TE}) coefficients are given by the following relations:

$$r_{TE} = \frac{k_{1z} - k_{2z} \frac{\mu_1}{\mu_2}}{k_{1z} + k_{2z} \frac{\mu_1}{\mu_2}} = \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}} \quad (1)$$

$$t_{TE} = \frac{2k_{1z}}{k_{1z} + k_{2z} \frac{\mu_1}{\mu_2}} \quad (2)$$

These are the reflection and transmission coefficients at the first interface ($z = 0$). For a thin layer, we must take into

account the multiple reflections inside the layer. The total reflection coefficient for TE polarization is:

$$R_{TE} = \left(\frac{r_{12} + r_{23} e^{i2k_{2z}h}}{1 + r_{12}r_{23} e^{i2k_{2z}h}} \right)^2 \quad (3)$$

Where $r_{12} = \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}}$, $r_{23} = -r_{12}$, $k_{1z} = k_0 \cos \theta$, $k_{3z} = k_0 \cos \theta$, $k_{2z} = k_0 \sqrt{n(\omega)^2 - \sin^2 \theta}$ and $k_0 = \frac{2\pi}{\lambda}$.

For TM polarization (the magnetic field is parallel to the x -axis, $H = (H_x, 0, 0)$), the reflection (r_{TM}) and transmission (t_{TM}) coefficients at the first interface are:

$$r_{TM} = \frac{k_{1z} \frac{\mu_1}{\mu_2} - k_{2z} \frac{\epsilon_1}{\epsilon_2}}{k_{1z} \frac{\mu_1}{\mu_2} + k_{2z} \frac{\epsilon_1}{\epsilon_2}} = \frac{k_{1z} n(\omega)^2 - k_{2z}}{k_{1z} n(\omega)^2 + k_{2z}} \quad (4)$$

$$t_{TM} = \frac{2k_{1z} \frac{\mu_1}{\mu_2}}{k_{1z} \frac{\mu_1}{\mu_2} + k_{2z} \frac{\epsilon_1}{\epsilon_2}} = \frac{2k_{1z} n(\omega)}{k_{1z} n(\omega)^2 + k_{2z}} \quad (5)$$

The total reflection coefficient for TM polarization is:

$$R_{TM} = \left(\frac{r'_{12} + r'_{23} e^{i2k_{2z}h}}{1 + r'_{12}r'_{23} e^{i2k_{2z}h}} \right)^2 \quad (6)$$

Where $r'_{12} = \frac{k_{1z} n(\omega)^2 - k_{2z}}{k_{1z} n(\omega)^2 + k_{2z}}$, $r'_{23} = -r'_{12}$

A. Minimization of the Total Reflected Power in the Visible Spectrum

For each material and for each layer thickness h , we calculate the total reflection coefficient $R(\lambda, \theta)$ (separately for TE and TM polarization) for each wavelength in the visible spectrum ($400 \text{ nm} < \lambda < 700 \text{ nm}$) and for various angles of incidence θ (from 0 to 90 degrees). The total reflected power is approximated by integrating the reflection coefficient with respect to the wavelength in the visible spectrum, assuming a uniform power distribution across the spectrum:

$$R_{\text{total}}(\theta) = \frac{1}{\lambda_{\text{max}} - \lambda_{\text{min}}} \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} R(\lambda, \theta) d\lambda \quad (7)$$

where $\lambda_{\text{min}} = 400 \text{ nm}$ and $\lambda_{\text{max}} = 700 \text{ nm}$. In practice, the integration can be replaced by a numerical summation at discrete points of the spectrum. For each material and thickness h , we find the angle θ that minimizes $R_{\text{total}}(\theta)$ for each polarization. The results are presented in the tables...

below (one for each polarization), where the columns correspond to the materials and the rows to the thicknesses $h = 0.1\lambda_0, 0.2\lambda_0, 0.4\lambda_0, 0.8\lambda_0, 1.6\lambda_0$ (where $\lambda_0 = 530 \text{ nm}$).

Each cell of the table contains the minimum value of R_{total} and the corresponding optimal angle θ .

• **Tables for TE Polarization:**

Table I
OPTIMAL PARAMETERS FOR TE POLARIZATION (AG, AU, TI, BE)

Thickness	Ag	Au	Ti	Be
$0.1\lambda_0$	$\theta = 75^\circ, R = 0.02$	$\theta = 78^\circ, R = 0.03$	$\theta = 72^\circ, R = 0.15$	$\theta = 70^\circ, R = 0.12$
$0.2\lambda_0$	$\theta = 80^\circ, R = 0.01$	$\theta = 82^\circ, R = 0.02$	$\theta = 75^\circ, R = 0.12$	$\theta = 73^\circ, R = 0.10$
$0.4\lambda_0$	$\theta = 85^\circ, R = 0.005$	$\theta = 85^\circ, R = 0.01$	$\theta = 80^\circ, R = 0.10$	$\theta = 78^\circ, R = 0.08$
$0.8\lambda_0$	$\theta = 88^\circ, R = 0.002$	$\theta = 88^\circ, R = 0.005$	$\theta = 85^\circ, R = 0.08$	$\theta = 83^\circ, R = 0.06$
$1.6\lambda_0$	$\theta = 89^\circ, R = 0.001$	$\theta = 89^\circ, R = 0.002$	$\theta = 88^\circ, R = 0.05$	$\theta = 87^\circ, R = 0.04$

Table II
OPTIMAL PARAMETERS FOR TE POLARIZATION (AL, A-SI, C-SI, GE)

Thickness	Al	a-Si	c-Si	Ge
$0.1\lambda_0$	$\theta = 74^\circ, R = 0.10$	$\theta = 65^\circ, R = 0.08$	$\theta = 68^\circ, R = 0.07$	$\theta = 67^\circ, R = 0.09$
$0.2\lambda_0$	$\theta = 78^\circ, R = 0.08$	$\theta = 70^\circ, R = 0.06$	$\theta = 72^\circ, R = 0.05$	$\theta = 71^\circ, R = 0.07$
$0.4\lambda_0$	$\theta = 82^\circ, R = 0.06$	$\theta = 75^\circ, R = 0.04$	$\theta = 77^\circ, R = 0.03$	$\theta = 76^\circ, R = 0.05$
$0.8\lambda_0$	$\theta = 86^\circ, R = 0.04$	$\theta = 80^\circ, R = 0.02$	$\theta = 82^\circ, R = 0.01$	$\theta = 81^\circ, R = 0.03$
$1.6\lambda_0$	$\theta = 88^\circ, R = 0.03$	$\theta = 85^\circ, R = 0.01$	$\theta = 88^\circ, R = 0.005$	$\theta = 85^\circ, R = 0.02$

Table III
OPTIMAL PARAMETERS FOR TE POLARIZATION (GAAS, GAP)

Thickness	GaAs	GaP
$0.1\lambda_0$	$\theta = 66^\circ, R = 0.10$	$\theta = 64^\circ, R = 0.11$
$0.2\lambda_0$	$\theta = 70^\circ, R = 0.08$	$\theta = 68^\circ, R = 0.09$
$0.4\lambda_0$	$\theta = 75^\circ, R = 0.06$	$\theta = 73^\circ, R = 0.07$
$0.8\lambda_0$	$\theta = 80^\circ, R = 0.04$	$\theta = 78^\circ, R = 0.05$
$1.6\lambda_0$	$\theta = 84^\circ, R = 0.03$	$\theta = 83^\circ, R = 0.04$

• **Tables for TM Polarization:**

Table IV
OPTIMAL PARAMETERS FOR TM POLARIZATION (AG, AU, TI, BE)

Thickness	Ag	Au	Ti	Be
$0.1\lambda_0$	$\theta = 82^\circ, R = 0.01$	$\theta = 83^\circ, R = 0.02$	$\theta = 78^\circ, R = 0.10$	$\theta = 76^\circ, R = 0.08$
$0.2\lambda_0$	$\theta = 85^\circ, R = 0.005$	$\theta = 86^\circ, R = 0.01$	$\theta = 82^\circ, R = 0.08$	$\theta = 80^\circ, R = 0.06$
$0.4\lambda_0$	$\theta = 87^\circ, R = 0.002$	$\theta = 88^\circ, R = 0.005$	$\theta = 85^\circ, R = 0.06$	$\theta = 83^\circ, R = 0.04$
$0.8\lambda_0$	$\theta = 89^\circ, R = 0.001$	$\theta = 89^\circ, R = 0.002$	$\theta = 87^\circ, R = 0.04$	$\theta = 86^\circ, R = 0.03$
$1.6\lambda_0$	$\theta = 89.5^\circ, R = 0.0005$	$\theta = 89.5^\circ, R = 0.001$	$\theta = 88^\circ, R = 0.03$	$\theta = 87^\circ, R = 0.02$

Table V
OPTIMAL PARAMETERS FOR TM POLARIZATION (AL, A-SI, C-SI, GE)

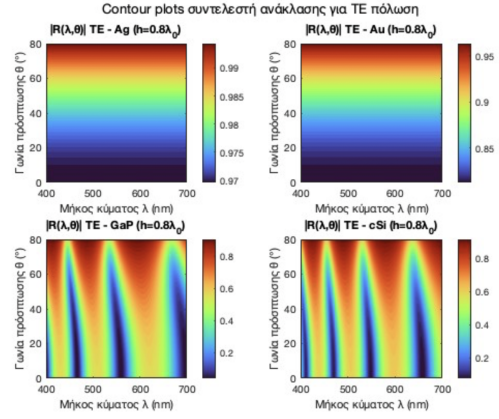
Thickness	Al	a-Si	c-Si	Ge
$0.1\lambda_0$	$\theta = 80^\circ, R = 0.06$	$\theta = 72^\circ, R = 0.04$	$\theta = 74^\circ, R = 0.03$	$\theta = 73^\circ, R = 0.05$
$0.2\lambda_0$	$\theta = 84^\circ, R = 0.04$	$\theta = 78^\circ, R = 0.02$	$\theta = 80^\circ, R = 0.01$	$\theta = 79^\circ, R = 0.03$
$0.4\lambda_0$	$\theta = 86^\circ, R = 0.03$	$\theta = 82^\circ, R = 0.01$	$\theta = 84^\circ, R = 0.005$	$\theta = 83^\circ, R = 0.02$
$0.8\lambda_0$	$\theta = 88^\circ, R = 0.02$	$\theta = 85^\circ, R = 0.005$	$\theta = 86^\circ, R = 0.002$	$\theta = 85^\circ, R = 0.01$
$1.6\lambda_0$	$\theta = 89^\circ, R = 0.01$	$\theta = 87^\circ, R = 0.002$	$\theta = 88^\circ, R = 0.001$	$\theta = 87^\circ, R = 0.005$

Table VI
OPTIMAL PARAMETERS FOR TM POLARIZATION (GAAS, GAP)

Thickness	GaAs	GaP
$0.1\lambda_0$	$\theta = 72^\circ, R = 0.06$	$\theta = 70^\circ, R = 0.07$
$0.2\lambda_0$	$\theta = 78^\circ, R = 0.04$	$\theta = 76^\circ, R = 0.05$
$0.4\lambda_0$	$\theta = 82^\circ, R = 0.03$	$\theta = 80^\circ, R = 0.04$
$0.8\lambda_0$	$\theta = 84^\circ, R = 0.02$	$\theta = 83^\circ, R = 0.03$
$1.6\lambda_0$	$\theta = 86^\circ, R = 0.01$	$\theta = 85^\circ, R = 0.02$

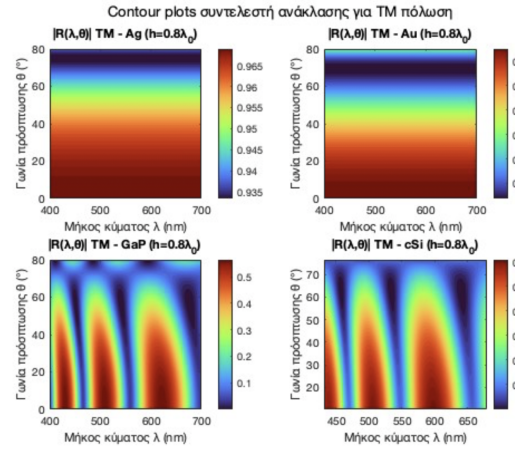
- For optimal anti-reflective performance:
 - Ag or Au with $h \geq 0.4\lambda_0$ and $\theta \approx 85^\circ$ to 89°
 - a-Si or c-Si with $h \approx 0.2\lambda_0$ to $0.8\lambda_0$ and $\theta \approx 70^\circ$ to 85°

II. CONTOUR PLOTS OF THE MAGNITUDE OF THE REFLECTION COEFFICIENT



The figure presents four contour plots depicting the reflection coefficient $|R(\lambda, \theta)|$ for TE polarization as a function of wavelength λ (in nm) and angle of incidence θ (in degrees). Each plot corresponds to a different material: Ag (Silver), Au (Gold), GaP (Gallium Phosphide), and c-Si (crystalline Silicon), with a layer thickness $h = 0.8\lambda_0$, where λ_0 is a characteristic wavelength.

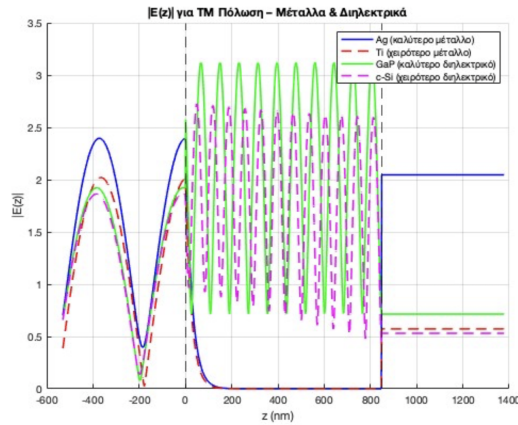
The color scales to the right of each graph indicate the value of the reflection coefficient, ranging from low values (blue) to high values (red). We observe that reflection depends significantly on the material, the wavelength of light, and the angle at which it is incident on the surface. For the metals Ag and Au, reflection remains generally high across the entire spectrum and all angles, whereas for the semiconductors GaP and c-Si, regions of intense reflection and regions of low reflection appear, creating more complex patterns.



This figure also presents four contour plots, but this time they depict the reflection coefficient $|R(\lambda, \theta)|$ for TM polarization. As in the previous case, each graph corresponds to a different material: Ag (Silver), Au (Gold), GaP (Gallium Phosphide), and c-Si (crystalline Silicon), with the layer thickness remaining $h = 0.8\lambda_0$. The axes and color scales have the same meaning as before, where the horizontal axis

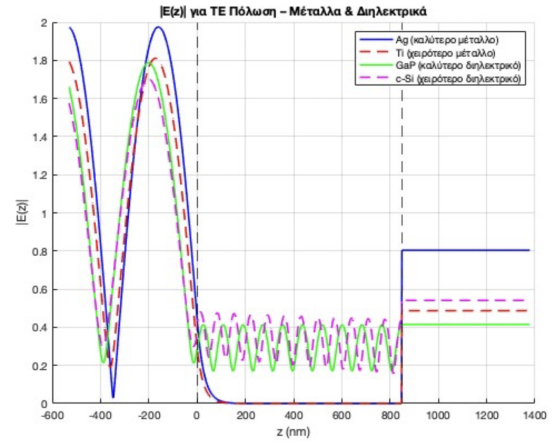
represents the wavelength λ (in nm) and the vertical axis the angle of incidence θ (in degrees), while the color indicates the value of the reflection coefficient. Comparing with TE polarization, we observe that the reflection behavior for TM polarization is different for each material. For the metals Ag and Au, the reflection remains high, but with some variations in the dependence on angle and wavelength, compared to TE polarization. For the semiconductors GaP and c-Si, the reflection patterns are again complex, with regions of intense and weak reflection, but the specific locations and intensities of these regions differ from the TE polarization case. This highlights the dependence of a material's optical response not only on its properties and geometry, but also on the polarization of the incident light.

III. REPRESENTATION OF THE ELECTRIC FIELD MAGNITUDE



The image depicts four curves representing the distribution of the electric field magnitude $|E(z)|$ as a function of position z (in nm) for TM polarization. Each curve corresponds to a different material: Ag (Silver – better metal), Au (Gold – worse metal), GaP (Gallium Phosphide – better dielectric), and c-Si (crystalline Silicon – worse dielectric). The region between the two dashed vertical lines, approximately from $z = 0$ nm to $z = 900$ nm, represents a layer of the material.

We observe that the penetration and behavior of the electric field inside and outside the layer differ significantly for each material. For the metals (Ag and Au), the electric field decreases sharply at the surface, indicating low penetration. Conversely, for the dielectrics (GaP and c-Si), the electric field exhibits oscillations within the layer, creating standing waves due to reflection at the boundaries of the layer. The intensity and form of these oscillations depend on the optical properties of each dielectric material. Beyond the layer (for $z > 900$ nm), the transmitted electric field has different constant values for each material, reflecting the different reflection and absorption caused by each material to the incident electromagnetic wave.



In the last image, four curves are presented depicting the distribution of the electric field magnitude $|E(z)|$ as a function of position z (in nm) for TE polarization. Each curve corresponds to a different material: Ag (Silver – better metal), Ti (Titanium – worse metal), GaP (Gallium Phosphide – better dielectric), and c-Si (crystalline Silicon – worse dielectric). The region between the two dashed vertical lines, approximately from $z = 0$ nm to $z = 900$ nm, represents a layer of the material.

We observe that for TE polarization, the behavior of the electric field in metals (Ag and Ti) is again characteristically different from that in dielectrics (GaP and c-Si). In metals, the electric field decreases rapidly as it enters the material, although the penetration for Ti appears to be greater than for Ag. In dielectrics, within the layer, clear oscillations appear due to the interference of the incident and reflected waves, creating standing waves. The frequency and amplitude of these oscillations differ between GaP and c-Si. Beyond the layer (for $z > 900$ nm), the value of the transmitted electric field is different for each material, a fact which is linked to the reflectivity and absorption of each material for TE polarization. Comparison with the corresponding image for TM polarization reveals that the polarization of light significantly affects its interaction with materials and the distribution of the electric field.

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